Spatiotemporal Evolution of Non-Equilibrium Electrostatic Waves associated with the Fugasity in a Plasma

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Abstract

Nonlinear dissipative electrostatic waves associated with the gravitational and external field effects of dust grains is shown in a non-equilibrium plasma with the fugasity. Spatiotemporal evolution of electrostatic waves propagating in the system is demonstrated by numerical simulation. It is shown that the nonlinear term of this equation forms the steepening and the dissipative effect damps the wave amplitude. Damping of the wave grows as the collision frequency increases. In the very low collision frequency, when the gravitational force becomes comparatively larger than the electrostatic force, the wave exhibits an unstable bahavior. On the other hand, when the collision frequency increases, i.e., the acceleration for the grains is comparatively larger, the behavior of the nonlinear wave is mild.

Key words : non-equilibrium plasma, dissipative electrostatic waves, spatiotemporal evolution, simulation, fugasity

1. Introduction

The existence of particulate contaminants in etching, sputtering and deposition processors remains a major problem in engineering and space plasmas. Numerous investigations have been done in various types of properties of plasma experiments [1-6] and theories [7-13]. Most of the theoretical studies of dusty plasmas have been reported in equilibrium state, since the mathematical treatment of non-equilibrium dusty plasmas is very difficult. In most cases, dust grains do not have well-defined mass, charge and size, and thus it is often quite difficult to understand plasmas having such dust grains or fine (ultrafine) particles. In actual situations, for example, in low pressure glow discharge plasmas, since electrons acquire a lot of energy from the external field and are accelerated by the electric field, their temperature is very high. However, since positive ions give their energy to neutral particles and they lose their energy, the temperature of ions becomes very low. In the plasma edge region, the density gradient exists. This means that such plasmas are in general non-equilibrium state, but the non-equilibrium gives rise to the various application, e.g., plasma etching by radicals and material processing due to the glow discharge. The non-equilibrium state varies its structure due to the processes such as the internal nonlinearity, the injection of the energy from the external device [14-16] and throw away the entropy. Such plasmas form the attraction for plasma physical phenomena. On the other hand, it is observed that the dust grain density is not fixed in time

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and space because the gravitational and electrostatic forces act on dust grains. Not many theoretical works on the effects of spatial and temporal evolution of the dust grain fluid including the density gradient and the gravitation of dust grains have been done in non-equilibrium dusty plasmas. Although Ref.17 applies the van der Waals force to the plasma, there is no consideration of the non-equilibrium and dissipative effects. The low frequency dust drift waves are treated by Shukla et. al., but they are not in consideration of the non-equilibrium [18]. In these studies, it is implied that there is an equilibrium state and the wave analysis is done on the assumption that the equilibrium state does not change on the time scale of the waves. However, it is noted that, in our investigation, the nonlinear wave of the initial state changes its state on the time scale of the wave.

The purpose of this paper is to investigate the spatiotemporal evolution of the density of the negatively-charged dust grain fluid to understand the properties of the plasma with the grain density gradient and the gravitational effect as an example of non-equilibrium dusty plasmas. The source of the dissipation is associated with the electron-dust Coulomb collision. Moreover we study the dependence of the wave on the collision frequency and the magnitude of the electric field. Therefore, we show that this open system is described by a new dissipative nonlinear equation, which is derived in this investigation and the solution of this equation is shown explicitly by numerical simulation.

2. Modeling

We consider the one-dimensional low frequency electrostatic motion of dust grains. The radius of a dust grain is assumed to be much less than the interparticle distance, the Debye length, and the wavelength. Thus the dust grains can be considered as heavy point masses. We don't study the dust-charge variation. In the vicinity of the plasma-sheath boundary, where the dust grain density varies, as an example, we need to consider the chemical potential $\mu_d=(\partial G/\partial n_d)_{T,P}$ for the variation of the density of dust grains, where the fixed temperature T and the fixed pressure P are assumed. The chemical potential is closely related with the fugasity of the real plasma considered here. We assume that the dust grains are the isothermal process. Here G denotes the Gibbs free energy provided that the temperature and pressure of dust grains are the constant. In this case, the dust grains are given a force to move from the position of the higher value of μ_d to that of the lower one. Although μ_d is fixed in equilibrium state (closed system), this is not constant in non-equilibrium state (open system). The condition of non-equilibrium plasmas is that $\partial\mu_d/\partial t\neq 0$ holds. In actual situations, it has been observed that the density of dust grains varies due to the electrostatic and gravitational forces and therefore the residence time of dust grains is limited in laboratory and space [13]. However, we have not yet observed the papers where the chemical potential is considered in non-equilibrium dusty plasmas. We therefore show a finding concerning the acceleration of dust grains in a non-equilibrium dusty plasma by introducing the chemical potential.

We assume that there is no velocity gradient, the diffusion velocity is quite small and the density of dust grains is weak in this plasma. Although nonlinear waves can affect the dust-charge, we assume the grain-charge to be constant for the treatment of our simple model. When we focus our attention only on the motion of dust grains, the system is described by the equations of continuity and the conservation of the flux density for the perturbation. The equation of continuity is

$$\frac{\partial n_d}{\partial t}+\frac{\partial(n_d v_d)}{\partial x}=0. \tag{1}$$

The conservation law of the dust grain density flux is described by

$$\frac{\partial(n_d v_d)}{\partial t}+\frac{\partial(n_d v^2{}_d)}{\partial x}=-\nu n_d v_d-n_d\frac{\partial \mu_d}{\partial x}-\left(g-\frac{Q_d E}{m_d}\right)\frac{\partial(n_d x)}{\partial x}, \tag{2}$$

where $n_d=n_d/n_{d0}$, v_d, ν, g, $Q_d=eZ_d$, E and m_d are the dust grain density, the dust velocity, the electron-dust Coulomb collision frequency, the gravitational acceleration, the grain charge, the electric field and the grain mass, respectively. Here n_{d0}, e and Z_d denote the equilibrium grain density, the magnitude of the electronic charge and the charge number of dust grains, respectively. The last two terms of the right-hand side of (2) are associated with the gravitational and electrostatic forces. The charge neutrality is $n_i=n_e+Z_d n_d$ in equilibrium, where n_i and n_e denote the positive ion and electron densities, respectively. The chemical potential of the system is

$$\mu_d=\mu_{d0}+\frac{k_B T_d}{m_d}\ln\left(\frac{f}{p_0}\right)=\mu_{d0}+v_{td}^2\ln\left(\frac{n_d}{n_{d0}}\right), \tag{3}$$

where μ_{d0}, k_B, $f=\gamma p$, γ, p_0 and $v_{td}=(k_B T_d/m_d)^{1/2}$ denote the initial chemical potential, the Boltzmann constant, the fugasity, the revised coefficient for the real plasma, the initial pressure and the dust thermal velocity, respectively. The fugasity f implies the pressure of the plasma with the same energy as the Gibbs energy of the real plasma. Equation (3) holds in the isothermal process. The dissipative or viscous effects originate from electron-neutral collisions (which usually dominate in a wide range of gas pressure), electron-dust Coulomb collisions, and the effects influence dust grain density flux variation. For an acoustic perturbation, v_d is related to n_d/n_{d0}, by

$$v_d\approx\frac{\omega}{k}\frac{n_d}{n_{d0}}\approx v_{ph}\frac{n_d}{n_{d0}}, \tag{4}$$

where v_{ph} denotes the phase velocity. Using (1), (3), (4) and replacing n_d/n_{d0} with n, we reduce (2) to

$$\frac{\partial^2 n}{\partial t^2}+\nu\frac{\partial n}{\partial t}-\frac{\partial^2}{\partial x^2}(v_{td}^2 n+v_{ph}^2 n^3)+\left(g-\frac{Q_d E}{m_d}\right)\frac{\partial^2(nx)}{\partial x^2}=0 \tag{5}$$

This equation is derived for the first time in non-equilibrium dusty plasmas, which depends on the wave velocities, collision frequency and the acceleration of the dust grains. It is noted that the coefficient of the last term of eq.(5) denotes the acceleration of the particulates. We simulate spatiotemporal evolution of eq.(5) numerically in the following section.

3. Simulation

In order to investigate the nonlinear properties of (5), we perform numerical calculation of this equation. Figure 1a shows the linear wave provided that the nonlinear term of (5)=0, where the parameters $T_e=1.0$ eV, the background plasma density $n_0=10^{16}\,\mathrm{m}^{-3}$, the dust grain density $n_d=10^8\,\mathrm{m}^{-3}$, $\nu=5$ Hz, $Q_d=eZ_d=1.6\times10^{-16}$ C, $Z_d=10^3$, the average dust grain mass $m_d\approx10^{-16}$ kg, the electric field $E=5.5$ V/m and $v_{td}=0.04$ m/s. In this case, the Debye length $\lambda_D=$

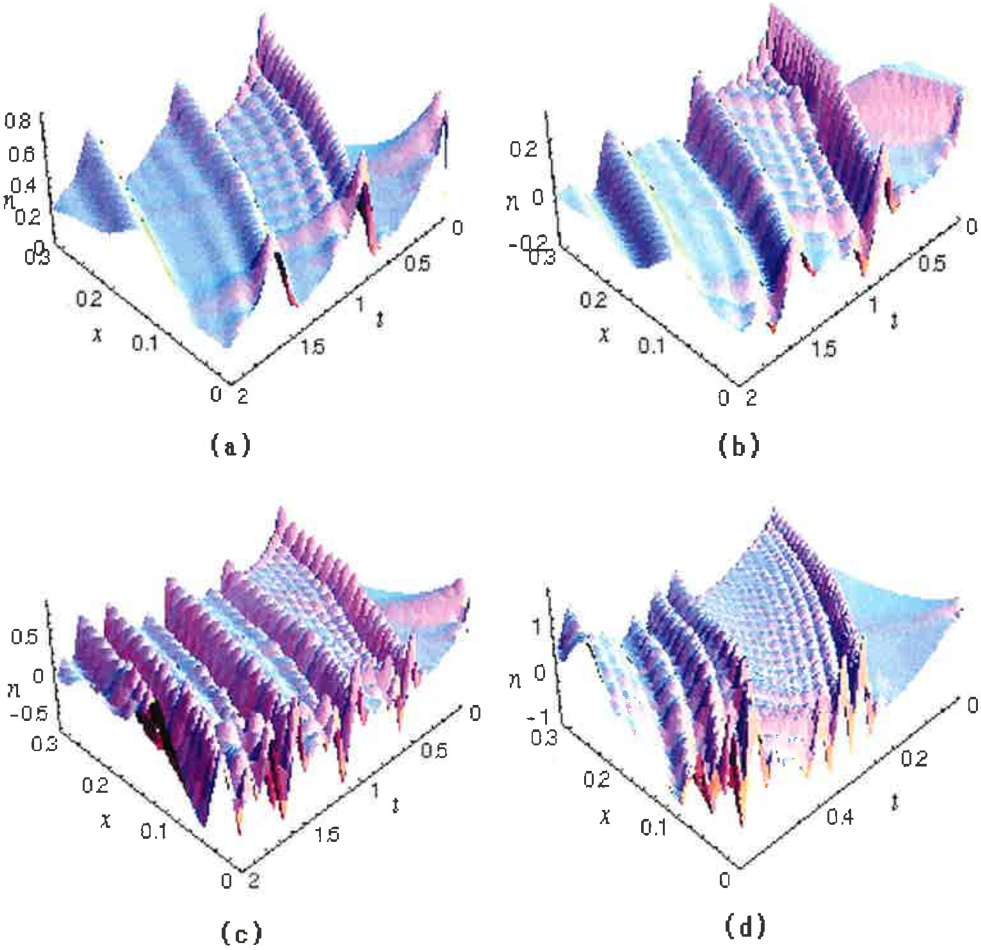


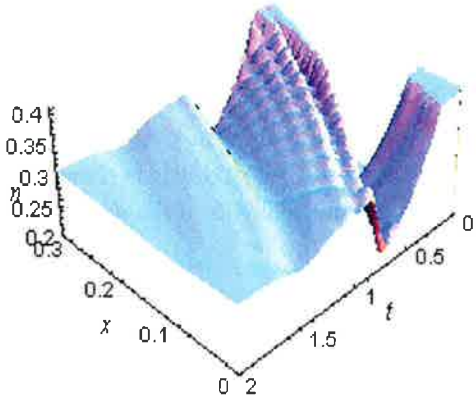
Figure 1a The linear solution of eq.(5), where $n_0=10^{16}\,\mathrm{m}^{-3}$, $n_d=10^8\,\mathrm{m}^{-3}$, $\nu=5$ Hz, $Q_d=eZ_d=1.6\times10^{-16}$ C, $Z_d=1000$, $m_d\approx10^{-16}$ kg, $E=5.5$ V/m and $v_{td}=0.04$ m, respectively.

Figure 1b Spatiotemporal-profile of eq.(5), where we used $v_{ph}=0.15$ m/s and the other parameters are the same as Fig. 1a.

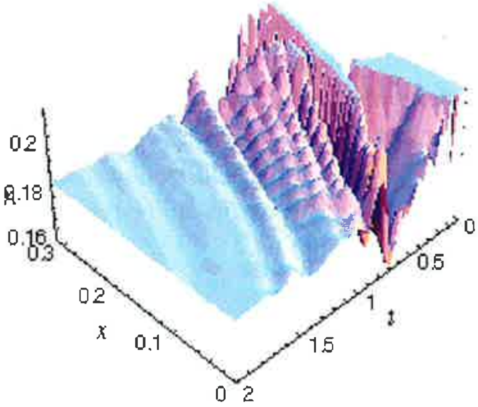
Figure 1c Spatiotemporal-profile in the case of $E=1.56$ V/m, where the parameters $Z_d=1000$, $\nu=5$ Hz, $v_{td}=0.04$ m/s and $v_{ph}=0.15$ m/s.

Figure 1d Spatiotemporal-profile of the case $E=0$, where the other parameters are the same as Fig. 1a.

7.4×10^{-5} m and the intergrain distance $n_d^{-1/3}=4.6\times10^{-4}$ m. Since $\lambda_D \ll n_d^{-1/3}$ and this implies that the grain density is not dense, we can assume that dust grains are distributed as the isolated particulates. The initial and the boundary conditions are $n(0, x)=\cos(-\omega x/v_{td})\exp(-\nu x/2v_{td})$ at $t=0$ and $n(t, 0)=n(t, 0.1\pi)$, respectively. Here we note that $\nu \ll \omega$. The wave trains are observed in the very low collision frequency. We show a numerical solution of the nonlinear equation (5) in Fig. 1b, where the phase velocity $v_{ph}=0.15$ m/s for $T_d=0.1$ eV and the average dust grain mass $m_d\approx10^{-16}$ kg. We understand that the nonlinear effect causes the steepening



(a)



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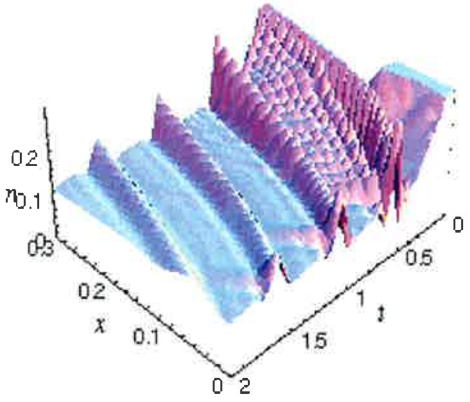
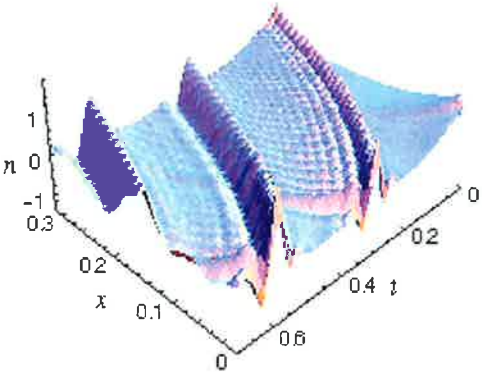


(c)



(d)

Figure 2a The linear wave when the collision frequency $\nu=10$ Hz, where the other parameters are the same as Fig. 1a.

Figure 2b Spatiotemporal-profile as the collision frequency $\nu=10$ Hz, where the other parameters are the same as Fig. 1b.

Figure 2c Spatiotemporal-profile of the wave of the case $\nu=10$ Hz, where the other parameters are as Fig. 1c.

Figure 2d Spatiotemporal-profile of the case $\nu=10$ Hz and $E=0$, where the other parameters are the same as Fig. 1d.

of the wave and the soliton-like structure is formed. It is observed that the nonlinear waves damp in time and space. When the electric field $E=2.5$ V/m, the dissipative nonlinear wave is shown in Fig. 1c. Since the effect of the gravitation becomes stronger than that of the electrostatic force, i.e., the acceleration due to the external forces for the grains becomes large, Fig. 1c implies that the damping of the wave is larger than that of the case of Fig. 1b. As an example, in the case $E=0$, the behavior of the wave becomes unstable as is seen in Fig. 1d.

Next we simulate a wave solution of (5) in the case of the higher collision frequency $\nu=$ 10 Hz. Figure 2a illustrates the linear wave profile in the case of the same parameters of Fig. 1a except for the collision frequency. We show a nonlinear wave in Fig. 2b, which corresponds to Fig. 1b. We observe the soliton-like structure due to the steepening of the wave. When the acceleration for the grains becomes large, the wave profile Fig. 2c doesn't vary so much and the amplitude of the nonlinear wave, which propagates in the negative direction of the x-axes, becomes unstable. In the case of no electric field, we show the nonlinear dissipative wave in Fig. 2d. The peak of the soliton-like structure grows and the wave instability is observed. It turns out that the wave varies in the region of $0<t<2.0$s and the behavior of the wave becomes unstable as time goes by.

4. Conclusion

We treat the nonlinear and dissipative behavior of dust grains in an open dusty plasma system by introducing the the fugasity associated with the chemical potential. We understand by numerical calculation that, in plasmas with the deviation from the equilibrium state, the nonlinear waves damp temporally and spatially due to the density gradient, the electron-dust Coulomb collision and the external forces. It is because that the first derivative of the left-hand side of (5) gives rise to the damping of the wave. The steepening is formed due to the competition between the nonlinear and dissipative effects and the soliton-like structure propagates in the negative x-direction accompanied with damping. The peak of oscillating nonlinear wave damps in time and space. Although the profile of the wave is mild in the case of the higher collision frequency, the soliton is formed due to the nonlinearity, but in the very low collision frequency the wave becomes unstable when the gravitational force is larger than the electric field. Since we use the plasma parameters employing the laboratory experiments, the findings obtained in this investigation could be able to apply to real plasmas. Although we only consider the chemical potential and our model is very simple, the damping or dissipative phenomenon of the dust grain density presented here may be useful in understanding the behavior of non-equilibrium dusty plasmas with the gravitation and the external field for dust grains in laboratory and space.

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