# The Temporal Evolution of the Grain-Charge on Electrostatic Waves in a Dusty Plasma

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## Abstract

The temporal evolution of the dust grain-charge and the influence of the ion density and temperature on the nonlinear waves in a dusty plasma are investigated by numerical calculation. Our results are in good agreement with that of the experiment on the charging process of dust grains and dust-acoustic waves. The nonlinear structure of the dust-charge is examined, and it is shown that the characteristics of the dust charge-number sensitively depend on the effects of the electrostatic potential, grain radius, ion density and temperature. It is found that the the nonlinear grain-charging sensitively depends on the ion to electron density ratio and the radius of the dust grain. New findings of variable-charge dust grains in a dusty plasma are predicted.

***Keywords***: Simulation, glow discharge plasma, grain charge, waves

## I. Intoroduction

The increase of recent interest in plasmas containing charged, micrometer-sized dust particles has arisen not only from the increase of observations of such plasmas in space environments such as cometary tails, planetary rings, and the lower ionosphere of the Earth[1-4], but also from their presence in laboratory devices[5-7]. In reality, the dust grains have variable-charge and mass due to fragmentation and coalescence. However, in studying collective effects involving charged dust grains in dusty plasmas one generally assumes that the dust particles behave like point charges. For low frequency nonlinear wave modes, the dust grains can be described as negative ions with large mass and large charge. Ion- and dust-acoustic wave modes in dusty plasmas have been treated by several authors[8-11]. We have suggested that high-speed streaming particles excite various kinds of nonlinear waves in space[12-14]. Dust grains are charged due to the local electron and ion currents, and its charge varies as a result of the change of the parameters such as the potential, densities, *etc*. Therefore, since the dust charge variation affects the characteristics of the collective motion of the plasma, the effect of the grain charge is of crucial importance in understanding dusty plasma waves. However, not many theoretical works on the charging process of dust grains have been done in dusty plasmas. In particular, the temporal evolution of the dust-charge has not been investigated in dusty plasmas.

In this paper, we focus our attention on the dust-charging on electrostatic waves in an

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unmagnetized dusty plasma. It is therefore instructive to examine the effects of the dust charging and ion temperature in dusty plasmas. Our plasma model consists of Boltzmann distributed electrons, positive ions, and the negatively charged dust fluid obeying the nonlinear continuity and momentum equations. We derive a nonlinear equation for variable-charge dust grains and the Sagdeev potential of electrostatic waves. We show the dependence of the grain-charging on the electrostatic potential, ion temperature and density. Our results show the existence of supersonic waves and illustrate the dependence of the dust-charge number on the parameters such as the potential, ion to electron density ratio and ion temperature.

In Sec. II, we present a new nonlinear equation for variable-charge dust grains and derive the Sagdeev potential from the basic equations. In Sec. III, we show the numerical results of the nonlinear equations obtained in the preceding section. It is shown that the grain-charge drastically changes due to the physical parameters. The simulation results are compared with the experimental results[15]. Section IV is devoted to the concluding discussion.

## II. Theory

We consider a collisionless, unmagnetized three component plasma consisting of Boltzmann electrons with a constant temperature $T_e$, warm ions having a temperature $T_i$ and negatively charged, heavy, dust particles, and assume that low frequency electrostatic waves propagate in this system. The number density of the electron fluid is assumed to be the Boltzmann distribution, $n_e = n_0 \exp(e\phi/T_e)$, where $n_e$, $n_0$, $e$ and $\phi$ are the electron density, background electron density, the magnitude of electron charge and the electrostatic potential.

The continuity equation and the equation of motion for ions are described by,

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}\left(n_i v_i\right) = 0, \tag{1a}$$

$$\left(\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x}\right) v_i + \frac{\gamma T_i}{m_i n_i}\frac{\partial n_i}{\partial x} + \frac{e}{m_i}\frac{\partial \phi}{\partial x} = 0, \tag{1b}$$

where $n_i$, $v_i$, $m_i$, $\gamma$ and $T_i$ denote the ion density, ion velocity, ion mass, specific heat ratio and ion temperature, respectively. Here, we express Eq. (1b) by the isothermal equation of state.

For one dimensional low frequency acoustic motions, we have the following two equations for the cold dust grains,

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}\left(n_d v_d\right) = 0, \tag{2a}$$

$$\left(\frac{\partial}{\partial t} + v_d \frac{\partial}{\partial x}\right) v_d - \frac{Q_d}{m_d}\frac{\partial \phi}{\partial x} = 0, \tag{2b}$$

where $n_d$, $v_d$ and $m_d$ refer to the dust grain density, dust fluid velocity and grain mass, respectively. Here the dust charge variable $Q_d = eZ_d$, where $Z_d$ is the charge number of dust grains measured in units of $e$.

The Poisson's equation is given as

$$\frac{\partial^2\phi}{\partial x^2}=\frac{e}{\varepsilon_0}\left(n_e-n_i+Z_d n_d\right). \tag{3}$$

We assume that the phase velocity of electrostatic ion waves is low in comparison with the electron thermal velocity. Charge neutrality at equilibrium requires that $n_{i0}=n_0+n_{d0}Z_d$, where $n_{i0}(n_{d0})$ denotes the equilibrium ion (dust grain) density. In this system, the ordering, $m_d \gg m_i \gg m_e$ holds, as is obtained in laboratory plasmas. Typical laboratory plasma frequencies are; $10^2$ Hz: $10^{5-6}$ Hz: $10^{9-10}$ Hz, and have roughly the same ordering as the mass ratios. Thus, the inclusion of the mass ratios is equal to considering the motion of dust particles.

We assume that the charge of the dust grain particles arises from plasma currents due to the electrons, ions and secondary electrons reaching the grain surface. In this case, the dust grain charge variable $Q_d$ is determined by the charge current balance equation[16]:

$$\left(\frac{\partial}{\partial t}+v_d\frac{\partial}{\partial x}\right)Q_d=I_e+I_i+I_s, \tag{4}$$

where $I_s$ denotes the secondary electron emission current. Assuming that the streaming velocities of the electrons and ions are much smaller than their thermal velocities, we have the following expressions for the electron and ion currents for spherical grains of radius $r$:

$$I_e=e\pi r^2(8T_e/\pi m_e)^{1/2}n_e(\phi)\exp\left(\frac{e\Phi}{T_e}\right), \tag{5a}$$

and

$$I_i=e\pi r^2(8T_i/\pi m_i)^{1/2}n_i(\phi,\ T_i)\left(1-\frac{e\Phi}{T_i}\right), \tag{5b}$$

where $\Phi=Q_d/r$ denotes the dust grain surface potential relative to the plasma potential $\phi$. If the ion streaming velocity $v_0$ is much larger than the ion thermal velocity, the ion current is approximately expressed as $I_i=e\pi r^2 v_0 n_i(1-2e\Phi/m_i v_0^2)$. At equilibrium, equating $I_e+I_i+I_s$ to zero we obtain the floating potential $\Phi_0$ and the equilibrium dust charge $Q_0=C\Phi_0$, where $C$ denotes the dust grain capacitance.

We normalize all the physical quantities as follows. The densities are normalized by the background electron density $n_0$. The space coordinate $x$, time $t$, velocities and electrostatic potential $\phi$ are normalized by the electron Debye length $\lambda_d=(\varepsilon_0 T_e/n_0 e^2)^{1/2}$, the inverse ion plasma period $\omega_i^{-1}=(\varepsilon_0 m_i/n_0 e^2)^{1/2}$, the ion sound velocity $C_s=(T_e/m_i)^{1/2}$, and $T_e/e$, respectively, where $m_i$, $\varepsilon_0$ and $e$ are the ion mass, the permittivity of vacuum and the magnitude of electron charge, respectively.

In order to study the temporal and spacial evolution of the dust grain-charge in non-stationary state, we obtain the equations $n_i=\delta_i/[1-2\phi/(M^2-\tau_i)]^{1/2}$ and $n_d=[(\delta_i-1)/Z_d]/[1+2\phi Z_d/\mu_d M^2]^{1/2}$ from (1a, b) and (2a, b), respectively where $\tau_i=T_i/T_e$. Here we used the boundary conditions, $\phi\to 0$, $n_d\to(\delta_i-1)/Z_d$, $n_i\to\delta_i$, $v_i\to 0$, $v_d\to 0$, at $\xi=x-Mt\to\infty$. From eq.(4), we derive a nonlinear equation for the charge of dust grains as

$$\left(\frac{\partial}{\partial t}+v_d\frac{\partial}{\partial x}\right)aZ_d=-\exp(\phi+aZ_d)+\frac{\delta_i(\tau_i-aZ_d)}{\sqrt{\mu_i\tau_i\left(1-\frac{2\phi}{M^2-\tau_i}\right)}}, \tag{6}$$

where $\mu_i=m_i/m_e$, $\mu_d=m_d/m_i$, $\tau_i=T_i/T_e$, $aZ_d=e\Phi/T_e$, $a=e^2/rT_e$ and the electron (ion) current $I_e(I_i)$ is normalized by $e\pi r^2(8T_e/\pi m)^{1/2}$. We assume that the specific heat ratio equals to 1, and $I_e+I_i\gg I_s$. Then, we can obtain the solution of (6) by numerical calculation in the next section.

Integration of the Poisson's equation (3) with the electron and ion densities *gives the Energy Law*, $(1/2)(\partial\phi/\partial\xi)^2+V(\delta_i,\phi)=0$. The Sagdeev potential $V(\delta_i,\phi)$ becomes

$$V(\delta_i,\phi)=1-\exp(\phi)+\delta_i(M^2-\tau_i)\left(1-\sqrt{1-\frac{2\phi}{M^2-\tau_i}}\right)+(\delta_i-1)\frac{\mu_d M^2}{Z_d}\left\{1-\sqrt{1+\frac{Z_d}{\mu_d}\frac{2\phi}{M^2}}\right\}. \tag{7}$$

The oscillatory solution of the nonlinear electrostatic waves exists when the following condition is satisfied. Nonlinear ion waves exist only when $V(\phi_M)\geq 0$, where the maximum potential $\phi_M$ is determined by $\phi_M=(M^2-\tau_i)/2$. The maximum Mach number and, correspondingly, the maximum amplitude of electrostatic ion waves significantly depends on the parameters $\delta_i$ and $\tau_i$.

## III. A simulation on the temporal evolution of the dust grain-charge

We examine the numerical calculation of the nonlinear equations obtained in the preceding section. In the following discussion, we assume that $r=10^{-7}$m, $T_e=1$ eV, $\mu_i=1836$ and $\mu_d=10^{12}$. For example a dust grain of radius 1 $\mu$m and mass density 2,000 kg/m$^3$ has a mass $\sim 5\times 10^{-15}$ kg so that $\mu_d\sim 10^{12}$. In order to solve (6) we use the finite difference method over the domains $0\leq x\leq x_{\max}$. The initial boundary conditions are given by $f(t=0,x)=f(t,x=0)=0$, $\partial f/\partial x=0$, at $x=x_{\max}$.

The result of an experiment of a direct current glow discharge[15] is summarized in Table 1. In the calculation the author evaluated using the relation $Z_d=Q_d/e$ that the dust-charge number $Z_d\sim 1{,}300$, where the grain radius $r=0.4$ $\mu$m and $\phi=-5.0$ V. However, $Z_d\sim 1{,}300$ is considered to be under estimation, because one can derive $Z_d\sim 1{,}390$ by using the same value of $r$ and $\phi$. Figure 1 is illustrated by using the experimental results (Table 1) as input parameters. It turns out that the the magnitude of the dust-charge increases with time and space coordinate increasing, and that finally the charge does not change in the range of $t>88$ $\mu$s and $x>1.47$ $\mu$m. In this stationary state, we obtain that the dust-charge $Z_d=1{,}422$. This result coincides with that of the experiment ($Z_d=1{,}390$) within the range of a few percent.

In order to study the collective effect of the dust-charge, we assume the input parameters as shown in Table 2. The temporal evolution of the dust-charge is plotted in Fig. 2 as a function of the ion density. As is discussed in previous studies, this effect occurs when the average intergrain distance $d(=n_d^{-1/3})$ becomes comparable to or less than the Debye length $\lambda_D$. Hence we estimate $d$ and $\lambda_D$ at the ion density $n_i=10^{15}$ m$^{-3}$ and $n_d=10^{11}$ m$^{-3}$, and we confirm

Table 1 Summary of the results of the experiment of Thompson et al.[15] on the grain-charge and dust-acoustic waves in a direct current glow discharge.

| | | |
|---|---|---|
| ($N_2$ plasma) | | |
| electron temperature | $T_e$ (eV) | : 2.5 |
| ion temperature | $T_i$ (eV) | : 0.03 |
| ion density | $n_i$ ($cm^{-3}$) | : $8\times10^8$ |
| ion mass | $m_i$ (g) | : $4.65\times10^{-23}$ |
| (dust particle) | | |
| temperature | $T_d$ (eV) | : 1.0 |
| density | $n_d$ ($cm^{-3}$) | : $2\times10^5$ |
| mass | $m_d$ (g) | : $6\times10^{-13}$ |
| radius | $r$ (cm) | : $4\times10^{-5}$ |

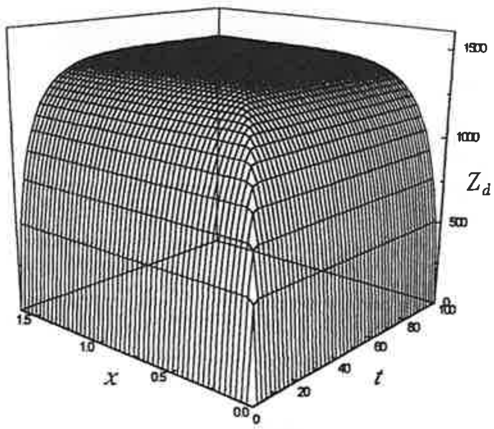


Fig. 1 The temporal and spacial evolution of the dust grain-charge. We use $\mu$s and $\mu$m as the temporal unit and spacial unit, respectively. The physical parameters used here are referred to Table 1.

Table 2 The input physical parameters used for numerical calculation.

| | | |
|---|---|---|
| ($N_2$ plasma) | | |
| electron temperature | $T_e$ (eV) | : 1.0 |
| ion temperature | $T_i$ (eV) | : 0.1 |
| ion density | $n_i$ ($cm^{-3}$) | : $1\times10^9$ |
| ion mass | $m_i$ (g) | : $4.65\times10^{-23}$ |
| (dust particle) | | |
| dust temperature | $T_d$ (eV) | : 1.0 |
| dust density | $n_d$ ($cm^{-3}$) | : $1\times10^5$ |
| dust mass | $m_d$ (g) | : $5\times10^{-12}$ |
| dust radius | $r$ (cm) | : $1\times10^{-4}$ |

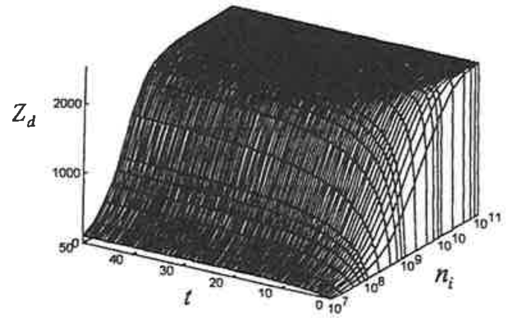


Fig. 2 The effect of the plasma density $n_i(10^{-6}\mathrm{m}^{-3})$ on the dust grain-charge $Z_d$ as a function of time $t\,(\mu\mathrm{m})$. The parameters used here are referred to Table 2.

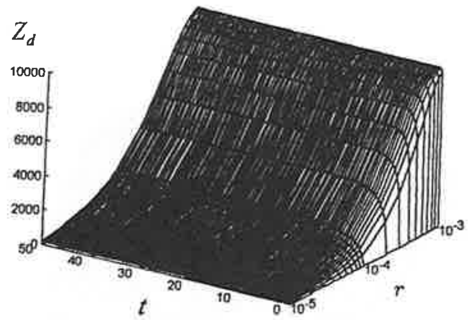


Fig. 3 The dust grain-charge $Z_d$ as a function of time $t\,(\mu\mathrm{s})$ and the radius of the grain $r$ $(10^{-2}\,\mathrm{m})$. The parameters used here are referred to Table 2.

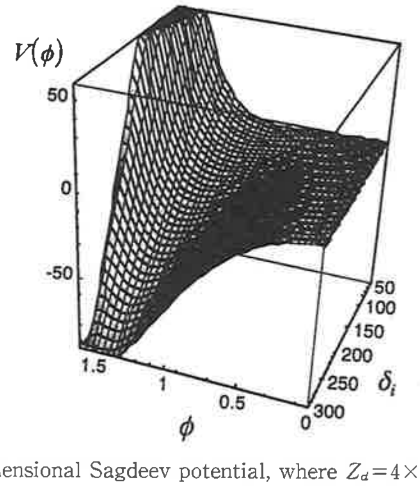


Fig. 4 A 3-dimensional Sagdeev potential, where $Z_d=4\times10^4$ and $M=1.8$.

that $d=2.2\times10^{-4}$ m is comparable to the Debye length $\lambda_D\approx2.3\times10^{-4}$ m. It is shown that, from Fig. 2, the saturation time decreases as the ion density increases. It is because the charge current increases with the ion density increasing as is shown in (6).

We expect that the grain radius affects the dust-charge as well as the electron density, because the parameter $a$ is proportional to $(rT_e)^{-1}$. In Fig. 3, we simulate the temporal evolution of the dust grain-charge when the grain radius changes. In the calculation, we vary the mass of the dust grains because $m_d$ is proportional to $r^3$. As is expected, Fig. 3 indicates that the dust-charge increases as the radius of the grain increases. Next, we show a 3-dimensional Sagdeev potential in Fig. 4 as a typical case where $Z_d=4\times10^4$ and $M=1.8$. We understand that the existence of nonlinear electrostatic waves changes according to the ion to electron density ratio.

Since the present model reproduces the experimental results, our results are important in understanding the charging process of dust grains.

## IV. Discussion

In this article, we have demonstrated the temporal and spacial evolution of the dust grain-charge and shown the existence of electrostatic waves in a dusty plasma whose constituents are electrons, ions and a cold dust fluid consisting of negatively-charged, micrometer-sized grains. Such plasmas may exist in both space environments and laboratory. We find the remarkable properties of the dust grain-charge obtained here as follows.

(1) The nonlinear temporal and spacial evolution of the grain-charge is shown in a dusty plasma. The simulation results presented here coincide with that of the experiment. Dependence of the dust grain-charge on the ion density and temperature is found in dusty plasmas.

(2) The effect of the dust-charge is of crucial importance in the sense that the dust charge number drastically changes due to the parameters such as the floating potential of dust grains, plasma potential, ion to electron density ratio, dust to ion mass ratio and ion temperature. The region for existence of nonlinear waves varies due to the ion to electron density ratio and floating potential of dust grains.

In a recent plasma experiment, it has been suggested that electrostatic waves may be responsible for the trapping of micrometer- and submicrometer-sized contamination particles within the plasma. As is mentioned in this article, the inclusion of impurity particles is required to consider the effects of the nonlinear dust-charging and waves. If such effects are present in dusty plasmas, which are observed in space environments and technological-aided plasmas, the investigation of their peculiar features will contribute to the future development in dusty plasmas. In this situation, our results are important in understanding the charging mechanism of dust grains and confirming the existence of electrostatic waves in dusty plasmas.

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## References

1 C.K. Goertz, Rev. Geophys. **27**, 1764 (1986).
2 E. Grün, G.E. Morfill and D.A. Mendis, in Planetary Rings (Univ. Arizona Press, Tucson, 1984), p. 275.
3 T.W. Hartquist, O. Havnes and G.E. Morfill, Fundam. Cosmic Phys. **15**, 107 (1992).
4 V.N. Tsytovich, and O. Havnes, Comm. Plasma Phys. Control. Fusion, **15**, 267 (1993).
5 R. Bingham, U. de Angelis, V.N. Tsytovich, and O. Havnes, Phys. Fluids B **3**, 811 (1991).
6 G.S. Selwyn, J.E. Heidenreich and K.L. Haller, Appl. Phys. Lett. **57**, 1876 (1990).
7 L. Boufendi et al., Appl. Phys. Lett. **60**, 169 (1992).
8 N.N. Rao, P.K. Shukla and M.Y. Yu, Planet. Space Sci. **38**, 543 (1990).
9 Y. Nejoh, IEEE Trans. Plasma Sci. **25**, 492 (1997).
10 A. Barkan, and R.L. Merlino, Phys. Plasmas **2**, 3261 (1995).
11 A. Barkan, R.L. Merlino and N. D'Angelo, Phys. Plasmas **2**, 3563 (1995).
12 Y. Nejoh, Phys. Plasmas **3**, 1447 (1996).
13 Y. Nejoh, Phys. Plasmas **4**, 2813 (1997).
14 Y. Nejoh, Nonl. Proc. Geophys. **5**, 53 (1998).
15 C. Thompson et al., Phys. Plasmas **4**, 2331 (1997).
16 F. Melandso, F.T. Askalsen and O. Havnes, Planet. Space Sci. **41**, 312 (1993).