Non-geodesic Equation of Motion in 3-dimensional Space: Towards the Correct Altitude-control of Bulky Space-crafts

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Abstract

It is shown that A. Einstein's equation of motion in a curved space-time continuum is not adequate to the problems containing the (gravitational) force. A possible new equation is derived for a 3-dimensional curved space, and this is also the generalization of geodesic. The general theory of differential geometry and the star-operator in harmonic integrals are used. The similarity of our result to "the dynamical system of gradient field" is pointed out. Our theory possibly gives the theoretical foundation for such a practical dynamical-problem as "Nonlinear dynamics of failure tolerant altitude control systems for space crafts," where they uses the ingenious gyroscope-technique. Our theory makes it possible to calculate the fictitious forces systematically in its simplest case.

Keywords: non-geodesic; equation of motion; 3-dimensional space; curved space; gradient field; altitude control; space craft

§ 1. Introduction

As is well known A. Einstein[5] has adopted the equation of geodesic in Riemannian geometry as the equation of motion of a point-particle in a curved space-time continuum. Geodesic is the generalization of the straight line in a Euclidean space and its tangent vectors are all parallel to each other in the well known meaning.[7] On the other hand it is well known that Newton's equation of motion gives the curve in a Euclidean space whose tangent vectors are generally not parallel to each other in a usual meaning, though they are parallel to each other in the case where the force-term is zero. Therefore geodesic is not the sufficient generalization of Newton's equation of motion, though it might be the generalization of the straight motion in a Euclidean space to a curved space-time continuum. The other concepts rather than parallel are needed for the generalization of Newton's equation. What kind of equation is the correct generalization of Newton's equation? The present paper gives an answer to this question for the case of a 3-dimensional curved space continuum on the basis of a general theory of differential geometry.[7,12,13,14]

The plan of the paper is the following: In Section 2 we construct a closed form α defined on a coordnate neighborhood U of an n-dimensional paracompact C^∞-manifold M from the

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