

The Boundary Value Problem of Laplace Equation and Newtonian Potential

— Weber Potential versus Coulomb Potential —

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Abstract

The electrostatic potential ϕ around a charged metal-disc is well known as Weber potential. The charge density of the disc is also well known. For this density we calculate the Newtonian (Coulomb type) potential V . It is proved that the potential V is a hyperfunction on the disc. On the other hand ϕ takes a constant value on the disc. This suggests the difference between V and ϕ .

§ 1. Introduction

It is well known that there are two methods for calculating the electrostatic potential of three dimensional problem: The method of boundary value problem of Laplace equation and that of the Newtonian (Coulomb type) potential V . However it is generally not aware of the fact that the two methods are mutually not obviously equivalent in some case. In this paper this fact is clarified.

In the next section (§ 2) we review the electrostatic potential ϕ around a charged metal-disc obtained by Weber as an example of the boundary value problem of Laplace equation. In § 3 we calculate the Newtonian potential V for Weber's charge density on the disc. In § 4 we show that $V(\rho, 0)$ is a hyperfunction^{2,7)} on the disc.

§ 2. Weber potential (A boundary value problem of Laplace equation)

Let us consider the electrostatic potential $\phi(\rho, z)$ around a charged metal-disc with radius a , of infinitesimally thin. Here $\rho, (\phi), z$ is a cylindrical coordinates with the axis of symmetry as its polar axis. This is a typical example of the boundary value problem of Laplace equation of three dimension. The potential $\phi(\rho, z)$ is well known³⁾:

$$\phi(\rho, z) = \frac{q}{a} \arcsin \left[\frac{2a}{\sqrt{(\rho-a)^2 + z^2} + \sqrt{(\rho+a)^2 + z^2}} \right] \quad (1)$$

where q is the total charge on the disc. From Eq. (1) we obtain the well known formula for the charge density on the disc³⁾:

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