

# 複素母数の完全楕円積分について

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## On the Complete Elliptic-Integrals with Complex Modulus

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### Abstract

The complete elliptic-integrals of 1st  $K(k)$  and 2nd kind  $E(k)$  are considered in complex  $k$ -plane. The domains in which they are analytic, are clarified. They can be considered as hyperfunctions for  $k \in (-\infty, -1]$  and  $[1, \infty)$ .

The complete elliptic-integrals of 1st  $K(k)$  and 2nd kind  $E(k)$  have been widely used in applied mathematics and physics. However, as far as we know, there is no article which states clearly the domain of their definition and the analytic property, concerning their complex moduli.

In this paper we clarify the domain in which  $K(k)$  and  $E(k)$  is analytic. Our conclusion is that both  $K(k)$  and  $E(k)$  can be considered as hyperfunctions<sup>(1,2)</sup> for  $k \in (-\infty, -1]$  and  $[1, \infty)$ .

Let us use the symbol

$$A := C \setminus \{\xi \in \mathbf{R} \mid \xi \leq -1 \text{ or } 1 \leq \xi\} \quad (1)$$

We can show that the complete elliptic-integral of 2nd kind,

$$E(k) := \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta} \, d\theta \quad (2)$$

converges in norm for any  $k$  belonging to any compact subset of  $A$ .

The essential points of the proof depend on the facts,

$$1 - k^2 \sin^2 \theta \in W \quad (3)$$

$$|\sqrt{1 - k^2 \sin^2 \theta}| \leq M \quad (4)$$

and

$$\int_0^{\pi/2} |\sqrt{1 - k^2 \sin^2 \theta}| \, d\theta \leq \frac{\pi}{2} M < \infty \quad (5)$$

for

$$0 < \theta < \frac{\pi}{2}, \quad k \in A' \quad (6)$$

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