

ION ACOUSTIC SOLITONS IN DISPERSIVE MEDIA

Yasunori NEJOH

We consider the two types of soliton which expressed ion acoustic wave in dispersive media. One of this is Korteweg-de Vries soliton in weak dispersive media and the other is envelope soliton associated with nonlinear Schrödinger equation in strong dispersive media. Two kinds of solitons were obtained by applying the reductive perturbation method.

I Introduction

Recently, solitons represented ion acoustic waves in plasma have been studied rapidly.⁽¹⁻⁴⁾ Especially, Washimi and Taniuti⁽²⁾ derived Korteweg-de Vries equation in weak dispersive media used Gardner and Morikawa transformation⁽⁴⁾ in order to express finite amplitude ion acoustic wave. This theoretical study is confirmed by experiments of Ikezi and his co-workers.^(5,6)

In (B) of the second section, we consider envelope soliton associated with the nonlinear Schrödinger equation in ion acoustic wave with strong dispersive media. Section three is devoted discussion.

II Ion acoustic waves in dispersive media

(A) weak dispersive effect (Korteweg-de Vries soliton)

Ion acoustic wave in weak dispersive media expressed following equation⁽²⁾,

$$\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial \xi} + \frac{1}{2} \frac{\partial^3 U}{\partial \xi^3} = 0 \quad (1)$$

This equation called Korteweg-de Vries equation and can be solved by means of the reductive perturbation method. Here U is the amplitude of wave, τ and ξ are time coordinate and space variable. It is well known that the steady solution of this equation is the following Korteweg-de Vries soliton,

$$U(\xi, \tau) = U_0 \operatorname{sech}^2 \left\{ (6U_0)^{\frac{1}{2}} (\xi - \lambda\tau - \alpha) \right\} \quad (2)$$

where λ is the velocity of soliton and is proportional to U . Ikezi and his co-workers succeeded the following experiment for above mentioned theoretical prediction. In effect, their found that a large amplitude perturbation breaks up solitons in the case of the external signal given to the circuit. And it is confirmed that the amplitude of soliton is proportional to the velocity. They succeeded in making many epoch-making experiments.

(B) strong dispersive effect (envelope soliton)

Ion acoustic wave in strong dispersive media represented by following equation,

$$i \frac{\partial \Psi}{\partial \tau} + p \frac{\partial^2 \Psi}{\partial \xi^2} + q |\Psi|^2 \Psi = 0 \quad (3)$$

This equation known as the nonlinear Schrödinger equation.

Here U, τ, ξ are the amplitude, time variable and space coordinate of the wave envelope, respectively, and i represents the imaginary unit. p and q are the dispersive coefficient and the nonlinear coupling coefficient. The two depends on the wave number.

The solution of nonlinear Schrödinger equation takes as follows,

$$\left. \begin{aligned} \psi(\xi, \tau) &= \psi_0 \exp(i\beta) \operatorname{sech} \gamma, \\ \beta &= v\xi - \frac{1}{2}(v^2 - \psi_0^2)\tau + \epsilon, \\ \gamma &= \psi_0(\xi - \delta - v\tau), \end{aligned} \right\} \quad (4)$$

where v is the velocity of solution, δ and ϵ are constant,

The solution of this form called envelope soliton.

The property of this equation can be easily understood by introducing the real function ρ and σ through

$$\psi = \rho^{\frac{1}{2}} \exp\left\{i \int^{\xi} \frac{\sigma}{2\rho} d\xi\right\} \quad (5)$$

Substituting this equation for ψ in Eq.(3) gives

$$\frac{\partial \rho}{\partial \tau} + \frac{\partial(\rho \sigma)}{\partial \xi} = 0 \quad (6)$$

$$\frac{\partial \sigma}{\partial \tau} + \sigma \frac{\partial \sigma}{\partial \xi} = 2pq \frac{\partial \rho}{\partial \xi} + p^2 \frac{\partial}{\partial \xi} \left\{ \rho^{-\frac{1}{2}} \frac{\partial}{\partial \xi} \left(\rho^{-\frac{1}{2}} \frac{\partial \rho}{\partial \xi} \right) \right\} \quad (7)$$

Hence the perturbation modulating a constant amplitude ρ_0 and phase σ_0

$$\begin{pmatrix} \rho \\ \sigma \end{pmatrix} = \begin{pmatrix} \rho_0 \\ \sigma_0 \end{pmatrix} + \begin{pmatrix} \delta \rho \\ \delta \sigma \end{pmatrix} \exp\{i(K\xi - \Omega\tau)\} \quad (8)$$

;that is

$$\Omega = \left\{ \sigma_0 \pm (-2pq\rho_0)^{\frac{1}{2}} \right\} K + O(K^3) \quad (9)$$

which is nonlinear dispersion equation. We assume that the wave number K of envelope is small.

If the product of p and q take the opposite signs, the plane wave is stable (bright soliton) because the square root part in Eq.(9) is positive(Fig. 1). On the contrary, if pq take the same signs, the plane wave cannot be stable for small perturbation(dark soliton) i.e, the envelope occurs instability (Fig. 2). This is called modulational instability.

$$pq < 0$$

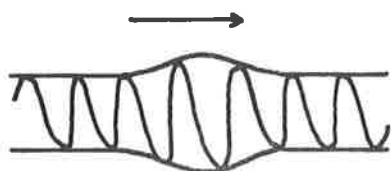


Fig. 1 bright soliton

$$pq > 0$$

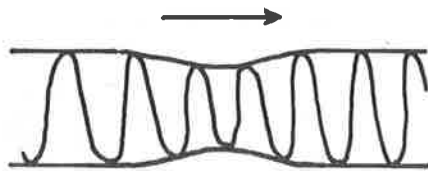


Fig. 2 dark soliton

III Discussion

There are several contradictions between theoretical prediction of Washimi and Taniuti and experiments of Ikezi et al..

One of them is how to make the theory which involves the effect of finite temperature. The trapping, the dissipation effect associated with nonlinear Landau damping by electrons as well as ions,⁹⁾ and the three wave interactions, must be considered simultaneously.

References

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